Binary Decision Diagrams in Reliability Theory

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Outline

1. Motivation
   - small example
   - stating the objectives

2. Introduction of BDDs

3. Evaluating effectiveness of BDDs
Temelín Nuclear Power Station – small example
Two types of analyses

Reliability analysis

- Qualitative analysis
- Quantitative analysis
Qualitative analysis

- we just know whether a component (or a whole system) works or does not work.
- it makes sense to consider components as boolean variables.

\[ x_i = \begin{cases} 
0, & \text{if the component is working,} \\
1, & \text{if the component is broken} 
\end{cases} \]
Then the functionality of the system consisting of (basic) components $x_1, \ldots, x_n$ is given by a boolean function, so-called structure function.

**Structure function $S$**

$$S(x_1, \ldots, x_n) = \begin{cases} 0, & \text{if the system is working}, \\ 1, & \text{if the system is broken} \end{cases}$$
Solution
Combination of component’s failures that can cause failure of the whole system.

Minimal solution
The smallest combination of component’s failures that can cause failure of the whole system.

Minimal solution set
Set of all minimal solutions.
Shannon’s decomposition

**Theorem (Shannon’s decomposition)**

For boolean function \( f = f(x_1, \ldots, x_n) \) and for \( i \in \{1, \ldots, n\} \) the following equality holds \( f = (x_i \land f\{x_i=1\}) \lor (\neg x_i \land f\{x_i=0\}) \).

- For example:

\[
(a \lor b) \land c = (a \land c) \lor (\neg a \land b \land c)
\]

- Graphically represented:
Let’s choose an ordering on variables and conduct Shannon’s decomposition in this order.
For our function $f(a, b, c) = (a \lor b) \land c$ we choose naturally $a < b < c$.
The result is a BDD:
After removing redundancies, the same function
\[ f(a, b, c) = (a \lor b) \land c \]
is more conveniently represented by:
For given boolean function, there is a unique RBDD representing it (when variable ordering is fixed)*.

Therefore, tautology test (well-known NP-complete problem) becomes trivial after obtaining RBDD representation of a function.

⇒ Construction of RBDD must be a NP-hard problem.

Utilization of RBDDs

(1) RBDD is a suitable representation of boolean function in computer memory.

(2) RBDDs have a lot of nice properties.

(3) RBDDs bring new options for reliability analysis.
   - We can find minimal solution set of a RBDD more effectively.
   - Afterwards, probability of system failure is trivial to compute.
Temelín Nuclear Power Station – small example

Binary Decision Diagrams in Reliability Theory
Minimal solutions

Minimal solution

The smallest combination of component’s failures that can cause failure of the whole system.

- Usually the main purpose of reliability analysis.
- Provide useful insight into complex or large systems.
- Make subsequent analyses much easier.
Advantages and disadvantages of BDDs

(+) theoretical comparison of BDDs with classic approaches is very hard, but experimental results are in favor of BDDs
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(− ? +) an efficient implementation is complex and requires careful memory representation
Examples used with a kind permission of Department of Risk Assessment, Temelín Nuclear Power Station